SOLUTIONS TUT. SHEET # 10

Q1.

Solution Let us consider the two corner voltages V_1 and V_1' that appear on the positive side of the waveform indicated in Fig. 2.8. This waveform represents the response of the high-pass filter when RC is very large compared to T. We can write that

$$V_1' = V_1 e^{-T/2RC}$$

$$V_2' = V_2 e^{-T/2RC}$$

$$V_1 - V_2' = V$$

$$V_1' - V_2 = V \qquad (1)$$

We can write another two equations due to the symmetry of the waveform.

$$V_1 = -V_2 \quad \text{and} \quad V_1' = -V_2'$$

$$V_1 - V_2' = V$$

Substituting for V_2', we have

$$V_1 - V_2 e^{-T/2RC} = V$$

$$V_1 - (V_1' - V)e^{-T/2RC} = V$$

$$V_1 - (V_1 e^{-T/2RC} - V)e^{-T/2RC} = V$$

$$V_1(1 - e^{-T/RC}) = V(1 - e^{-T/2RC})$$

When $x \ll 1$, we can write $e^{-x} \approx (1 - x)$ and $(1 - x)^{-1} \approx (1 + x)$. This relationship can be used when $T/2RC \ll 1$.

We can write

$$V_1 = \frac{V(1 - e^{-T/2RC})}{(1 - e^{-T/RC})} = \frac{V}{(1 + e^{-T/2RC})} \qquad (2)$$

Similarly we can also obtain that

$$V_1' = V_1 e^{-T/2RC} = \frac{Ve^{-T/2RC}}{(1 + e^{-T/2RC})} = \frac{V}{(1 + e^{+T/2RC})} \qquad (3)$$

Q2.

(a) $V_1' = V_1 e^{-T_1/RC} = V_1 e^{-0.1/0.2} = 0.606V_1$

$V_2 = V_1' - V = V_1' - 1 = 0.606V_1 - 1$

$V_2' = V_2 e^{-T_2/RC} = V_2 e^{-0.2/0.2} = 0.367V_2$

$V_1 = V_2' + V = V_2' + 1 = 0.367V_2 + 1$

We obtain the following values by solving the four equations.

$V_1 = 0.816$ V, $V_1' = 0.496$ V, $V_2 = -0.504$ V and $V_2' = -0.184$ V.

(b) Let positive area be A_1, and negative area be A_2.

$$A_1 = \int_0^{0.1} 0.816 e^{-t/0.2}\, dt = (-0.816(0.2)e^{-t/0.2})_0^{0.1} = 0.06 \text{ V-s}$$

$$A_2 = \int_0^{0.2} (-0.504) e^{-t/0.2}\, dt = (0.504(0.2)e^{-t/0.2})_0^{0.2} = -0.06 \text{ V-s}$$

$A_1 + A_2 = 0.06 + (-0.06) = 0$

The net area is obviously zero. This means that the series capacitor blocks the dc component of the inpu waveform.

Q3.

Solution The peak-to-peak amplitude of the symmetrical square-wave V is given as 2 V. We know that for a symmetrical square-wave, the average value is zero.

The waveform is transmitted through an RC low-pass filter. The rising portion of the output waveform is given by Eq. (2.52).

$$v_{o1}(t) = V' + (V_1 - V')e^{-t/RC} \quad (1)$$

We are dealing with a symmetrical square-wave with zero average value. We are given that $V' = -V'' = 1$ V. Let us assume that $T_1 = T_2 = t_p$. Substituting this in Eq. (1), we have

$$V = 1 + (-1 - V)\, e^{-t_p/RC}$$

$$V = 1 + (-1 - V)e^{-1} = 1 - (1 + V)(0.368)$$

$$1.368V = 0.632$$

$$V = \frac{0.632}{1.368} = 0.463 \text{ V}$$

Peak-to-peak voltage of the output is $2V$ that is equal to

$$v_o\text{(peak)} = 2V = 2 \times 0.463 = 0.926 \text{ V}$$

Q4.

Solution In the case of a symmetrical square waveform, we have $T_1 = T_2 = T/2$ and $V' = -V'' = V/2$. We also observe that $V_1 = -V_2$ due to symmetry. Substituting these values in Eqs (2.52) and (2.53), we can write

$$V_2 = \frac{V}{2} + \left(V_1 - \frac{V}{2}\right)e^{-T/2RC}$$

Since we know that $V_1 = -V_2$, we can write

$$V_2 = \frac{V}{2} + \left(-V_2 - \frac{V}{2}\right)e^{-T/2RC}$$

$$V_2(1 + e^{-T/2RC}) = \frac{V}{2}(1 - e^{-T/2RC})$$

$$V_2 = \frac{V}{2}\frac{(1 - e^{-T/2RC})}{(1 + e^{-T/2RC})}$$

Multiplying the numerator and denominator in the above equation with $e^{T/2RC}$, we have

$$V_2 = \frac{V}{2}\frac{(e^{T/2RC} - 1)}{(e^{T/2RC} + 1)}$$

If we introduce $x = T/4RC$, the above equation assumes a simple form as

$$V_2 = \frac{V}{2}\frac{(e^{2x} - 1)}{(e^{2x} + 1)}$$

$$V_2 = \frac{V}{2}\frac{(e^{x} - e^{-x})}{(e^{x} + e^{-x})}$$

$$V_2 = \frac{V}{2}\frac{(e^{x} - e^{-x})/2}{(e^{x} + e^{-x})/2} = \frac{V}{2}\frac{\sinh x}{\cosh x}$$

$$V_2 = \frac{V}{2}\tanh x$$

Q5

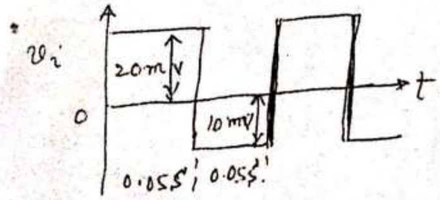

Fig.1.

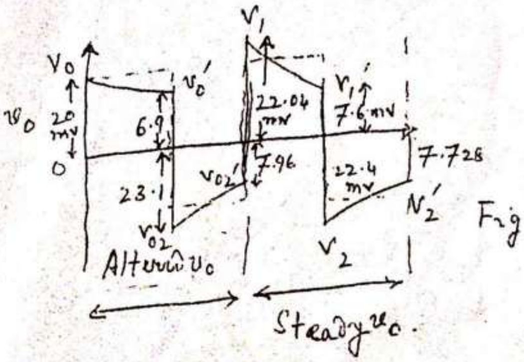

Fig.2.

Here with $v_0 = 20\,mV$, $RC = 0.47s$.

$v_0' = 20 \times 10^{-3} e^{-\frac{0.05}{0.047}} = 6.9 \times 10^{-3}$

$v_0' - v_{02} = 30 - 6.9 = 23.1$ ms.

$v_{02}' = v_{02} e^{-t/RC} = v_{02} \times 0.345$

$\therefore e^{-t/RC} = e^{-\frac{0.05}{0.047}} = 0.345$

$v_{02}' = 7.96$ mV

$\therefore v_1 = 30 - 7.96 = 22.04$ mV.

$v_1' = 22.04 \times 0.345 = 7.6$ mV.

$v_2 = 30 - 7.6 = 22.4$ mV.

$v_2' = 22.4 \times 0.345 = 7.728$ mV

After a few cycles v_2' stabilizes to 7.6 mV. The required v_0 is the steady v_0 shown in Fig.2

Q6. & Q7 These are theoretical questions (Given in the Book Milliman & Taub)